Class 5. The IS-LM model and Aggregate Demand

1. Use the Keynesian cross to predict the impact of:
   a) An increase in government purchases.
   b) An increase in taxes.
   c) An equal increase in government purchases and taxes.

2. In the Keynesian cross, assume that the consumption function is given by
   \[ C = 200 + 0.75(Y - T) \]
   Planned investment is 100; government purchases and taxes are both 100.
   a) Graph planned expenditure as a function of income.
   b) What is the equilibrium level of income?
   c) If government purchases increase to 125, what is the new equilibrium income?
   d) What level of government purchases is needed to achieve an income of 1,600?

3. Suppose the tax system is
   \[ T = \bar{T} + tY \]
   where \( \bar{T} \) is a fixed level of taxation and \( t \) is the marginal tax rate.
   a) How does this tax system change the way consumption responds to changes in GDP?
b) In the Keynesian cross, how does this tax system alter the government-purchases multiplier?

c) In the IS-LM model, how does this tax system alter the slope of the IS curve?

4. Consider the impact of an increase in thriftiness in the Keynesian cross. Suppose the consumption function is

\[ C = \bar{C} + c(Y - T) \]

where \( \bar{C} \) is autonomous consumption and \( c \) is the marginal propensity to consume.

a) What happens to equilibrium income when the society becomes more thrifty, as represented by a decline in \( \bar{C} \)?

b) What happens to equilibrium saving?

c) Why do you suppose this result is called the paradox of thrift?

d) Does this paradox arise in the classical model? Why or why not?

5. Suppose the money demand function is

\[ \left( \frac{M}{P} \right)^d = 1,000 - 100r \]

where \( r \) is the interest rate in percent. The money supply is 1,000 and the price level is 2.

a) Graph the supply and demand for real money balances.

b) What is the equilibrium interest rate?
c) Assume that the price level is fixed. What happens to the equilibrium interest rate if the supply of money is raised from 1,000 to 1,200?

d) If the Fed wishes to raise the interest rate to 7 percent, what money supply should it set?

6. Use the IS-LM model to predict the effects of each of the following shocks on income, the interest rate, consumption and investment. In each case, explain what the Central Bank should do to keep income at its initial level.

a) After the invention of a new high-speed computer chip, many firms decide to upgrade their computers systems.

b) A wave of credit card fraud increases the frequency with which people make transactions in cash.

c) A best seller titled *Retire Rich* convinces the public to increase the percentage of their income devoted to saving.
7. Consider the following economy:

\[ C = 200 + 0.75(Y - T) \]
\[ I = 200 - 25r \]
\[ \left( \frac{M}{P} \right)^d = Y - 100r \]
\[ M^s = 1,000 \]
\[ P = 2 \]
\[ \overline{G} = \overline{T} = 100 \]

a) Graph the IS curve for \( r \) ranging from 0 to 8.
b) Graph the LM curve, also for \( r \) ranging from 0 to 8.
c) Find the equilibrium interest rate and the equilibrium level of income \( Y \).
d) Suppose the government purchases are increased from 100 to 150. How much does the IS curve shift? What are the new equilibrium \( r \) and \( Y \)?
e) Suppose instead that the money supply is raised from 1,000 to 1,200. How much does the LM curve shift? What are the new equilibrium \( r \) and \( Y \)?
f) With the initial values for monetary and fiscal policy, suppose that the price level rises from 2 to 4. What happens? What are the new equilibrium \( r \) and \( Y \)?
g) Derive and graph an equation for the aggregate demand curve. What happens to this aggregate demand curve if fiscal or monetary policy changes as in questions d) and e)?

8. Explain why each of the following statements is true. Discuss the impact of monetary and fiscal policy in each of these special cases and state how the shape of the aggregate demand curve is affected.

a) If investment does not depend on the interest rate, the IS curve is vertical.
b) If money demand does not depend on the interest rate, the LM curve is vertical.
c) If money demand does not depend on income, the LM curve is horizontal.
d) If money demand is extremely sensitive to the interest rate, the LM curve is horizontal.

9. Suppose the government wants to raise investment but keep output constant. In the IS-LM model, what mix of monetary and fiscal policy will achieve this goal? In the early 1980s the US government cut taxes and run a budget deficit while the Fed (the US Central Bank) pursued a tight
monetary policy. What effect should this policy mix have?

10. The Central Bank is considering two alternative monetary policies: either holding the money supply constant and letting the interest rate adjust; or adjusting the money supply to hold the interest rate constant. In the IS-LM model, which policy will better stabilize output under the following conditions?

a) All shocks to the economy arise from exogenous changes in the demand for goods and services.

b) All shocks to the economy arise from exogenous changes in the demand for money.

11. Suppose that the demand for real money balances depends on disposable income; that is,

\[
\left( \frac{M}{P} \right)^d = L[r, (Y - t)]
\]

Using the IS-LM model, discuss whether this change in the money demand function alters the following:

a) The analysis of changes in government purchases.

b) The analysis of changes in taxes.
Answers

1.
   a) $\Delta Y \Rightarrow \Delta C$
   b) $\nabla C \Rightarrow \nabla Y$
   c) $\Delta Y \Rightarrow \Delta C$; also $\Delta Y = \Delta G = \Delta T$.

2.
   a) $E = C + I + G = 200 + 0.75(Y - 100) + 100 + 100$
      $E = 325 + 0.75Y$

   b) $E = Y$; $Y = 325 + 0.75Y$; $Y = 1,300$
   c) This effect can be solved using the multiplier.

\[
\Delta Y = \frac{1}{1 - 0.75} \Delta G
\]

$\Delta Y = 4 \times 25 = 100$

$\therefore Y' = Y + \Delta Y = 1,300 + 100 = 1,400$
d) \[ Y = 200 + 0.75(Y - 100) + 100 + G \]
\[ 1,600 = 200 + 0.75(1,600 - 100) + 100 + G \]
\[ G = 175 \]

3.

a)
\[ C = \bar{C} + c(Y - T) \quad \text{where} \quad T = \bar{T} + tY \]
\[ C = \bar{C} + c \left[ Y - (\bar{T} + tY) \right] \]
\[ C = (\bar{C} - c\bar{T}) + c(1 - t)Y \]
\[ \frac{dC}{dY} = c(1 - t) \]

\[ \therefore \] It lowers the marginal propensity to consume from \( c \) to \( c(1 - t) \)

b)
\[ G \text{ Multiplier} = \frac{1}{1 - MPC} = \frac{1}{1 - c(1 - t)} \]

Since the MPC has decreased, the denominator is higher and therefore the multiplier is smaller.

c) \( Y \) is now less responsive to changes in \( G \). Therefore the slope of the IS curve will be steeper in the \( r,Y \) plane. Check this answer for a specific example of the the IS curve. See that if \( I = I(r) \), the slope of the IS curve is
\[ \frac{dY}{dr} = \frac{I'(r)}{1 - c(1 - t)} \quad \text{or} \quad \frac{dr}{dY} = \frac{1 - c(1 - t)}{I'(r)} \]
If $t > 0$ the last expression will be larger (IS steeper) in absolute terms.

4.

a) Equilibrium income is

$$Y = \frac{1}{1-c} \left( \bar{C} + \bar{I} + \bar{G} \right) - \frac{c}{1-c} T$$

$$\frac{dY}{dC} = \frac{1}{1-c}; \quad dY = \frac{1}{1-c} d\bar{C}$$

Therefore, if $d\bar{C} < 0 \implies dY < 0$

Income will go down by the extent of the decrease in autonomous consumption times the multiplier.

b) 

$$S = Y - C - \bar{G} = Y - \bar{C} - c(Y - T) - \bar{G}$$

$$S = Y(1-c) - \bar{C} + cT - \bar{G}$$

$$\frac{dS}{dC} = (1-c) \frac{dY}{dC} - 1 = \frac{(1-c)}{(1-c)} - 1 = 0$$

Therefore, equilibrium saving is not affected by the increase in thriftiness.

c) Because saving does not increase despite that people consume less.

d) No, because in that model income is fixed.

There, as we saw in a previous problem,

$$dS = -d\bar{C}$$
5. a) 

\[
\left( \frac{M}{P} \right)^d = 1,000 - 100r
\]

\[
r = 10 - 0.01 \left( \frac{M}{P} \right)^d
\]

\[
\left( \frac{M}{P} \right)^s = \frac{1,000}{2} = 500
\]

b) \( r = 10 - 0.01(500) = 5 \)

c) \( r = 10 - 0.01(600) = 4 \)

d) \( 7 = 10 - 0.01(M / P)^s; \ (M / P)^s = 300 \)
6.

a) \( \Delta r, \Delta Y, \Delta C, \Delta I \)

The final increase in investment is smaller than the initial shock due to the crowding out effect of the rise in the interest rate.

b) \( \Delta r, \nabla Y, \nabla C, \nabla I \)

\( \Delta r, \nabla Y, \nabla C, \nabla I \)

c)
7.a) **IS curve**  
\[ Y = C + I + G = 200 + 0.75(Y - 100) + 200 - 25r + 100 \]
\[ Y(1 - 0.75) = 425 - 25r \]
\[ r = 17 - 0.01Y \]
\[ r = 0 \Rightarrow Y = 1,700 \]
\[ r = 8 \Rightarrow Y = 900 \]
b) **LM curve**

\[
\left( \frac{M}{P} \right)^s = L(Y, r)
\]

\[
\frac{1,000}{2} = Y - 100r
\]

\[
r = -5 + 0.01Y
\]

\[
r = 0 \Rightarrow Y = 500
\]

\[
r = 8 \Rightarrow Y = 1,300
\]

c) \[\begin{align*}
IS & \quad r = 17 - 0.01Y \\
LM & \quad r = -5 + 0.01Y
\end{align*}\] \[r=6; \quad Y=1,100\]
d) This produces a horizontal shift to the right in the IS curve that equals the change in $G$ times the multiplier. That is,

$$\Delta Y = \frac{1}{1 - 0.75} \Delta G = 4 \times 50 = 200$$

Therefore, the new IS curve is

$$Y = 1,900 - 100r \quad \text{or} \quad r = 19 - 0.01Y$$

The new equilibrium is

$$\begin{align*}
  r &= 19 - 0.01Y \\
  r &= -5 + 0.01Y
\end{align*}$$

\[ r = 7; \quad Y = 1,200 \]

\[ r = 7; \quad Y = 1,200 \]

e) This produces a downward shift in the LM curve. Draw the graph. The new equilibrium is given by,
\[ r = 17 - 0.01Y \quad r = -6 + 0.01Y \]

\( r = 5.5; \quad Y = 1,150 \)

f)
The IS curve stays the same. The LM curve changes to
\[ \frac{1,000}{4} = Y - 100r \]
\( r = -2.5 + 0.01Y \)

New equilibrium,
\[ r = 17 - 0.01Y \quad r = -2.5 + 0.01Y \]
\( r = 7.25; \quad Y = 975 \)

\[ \Delta P \Rightarrow \nabla \frac{M}{P} \Rightarrow \Delta r \Rightarrow \nabla I \Rightarrow \nabla Y \]
g) \[
\begin{align*}
IS & \quad Y = 1,700 - 100r \\
LM & \quad \frac{1,000}{P} = Y - 100r
\end{align*}
\]

Eliminate \( r \). For instance, solve the first equation for \( r \) \((r = 17 - 0.01Y)\) and substitute the result on the second equation to get,

\[
\frac{1,000}{P} = Y - 1700 + Y
\]

Then, solve for \( Y \)

\[
Y = 850 + \frac{500}{P}
\]

This is the Aggregate Demand curve.

To graph this curve note that

\[
P \to \infty \quad Y \to 850
\]

\[
P \to 0 \quad Y \to \infty
\]

So, the graphical representation of this curve is,
Keeping explicit $G$ and $M$, the aggregate demand curve can be written as follows (Check that you understand this):

$$Y = 650 + 2G + \frac{1}{2} \frac{M}{P}$$

It is easy to see that for the initial values of $G$ and $M$ (100 and 1,000) this equation reduces to

$$Y = 850 + \frac{500}{P}$$

which is the original Aggregate Demand (AD) curve.
Now, if there is an increase in government expenditure to 150, we find that the new AD curve is

\[ Y = 950 + \frac{500}{P} \]

That is, the new AD curve will be asymptotic to \( Y=950 \). The AD will shift in a parallel fashion to the right by 100.

If money supply goes to 1,200, then the AD curve is

\[ Y = 850 + \frac{600}{P} \]
The asymptotic values of the curve do not change, but the slope does and the result is that for the interest rate values considered here, the effect is a rightward shift of the AD curve.

8. To solve this problem, the best approach is to work with an explicit formal representation of the IS-LM model and AD curve, such as the one considered in the Appendix to Chapter 11 of Mankiw.
To remind ourselves,
\[ C = a + b(Y - T) \]
\[ I = c - dr \]
\[ L(r, Y) = eY - fr \]
\( a, b, c, d, e \) and \( r \) are all positive parameters.

**IS curve:**
\[
Y = \frac{a + c}{1-b} + \frac{1}{1-b}G - \frac{b}{1-b}T - \frac{d}{1-b}r
\]

**LM curve**
\[
r = \frac{e}{f}Y - \frac{1}{f} \left( \frac{M}{P} \right)
\]

**Aggregate Demand curve**
\[
Y = \frac{z(a + c)}{1-b} + \frac{z}{1-b}G - \frac{zb}{1-b}T + \frac{d}{(1-b)} \left[ \frac{M}{f + de/(1-b)} \right]
\]

Where,
\[
z = \frac{f}{[f + de/(1-b)]}
\]

a) **Investment does not depend on \( r \) \((d=0)\)**

If \( d=0 \), \( z=1 \)

**IS**
\[
Y = \frac{a + c}{1-b} + \frac{1}{1-b}G - \frac{b}{1-b}T \quad \text{; therefore IS vertical}
\]
AD \ Y = \frac{a + c}{1-b} + \frac{1}{1-b} G - \frac{b}{1-b} T \\

Therefore \ AD \ \text{vertical}. \ AD \ does \ not \ react \ to \ monetary \ policy. \ Only \ to \ fiscal \ policy.

b) \ Money \ demand \ does \ not \ depend \ on \ r \ (f=0) \\

If f=0, z=0

Multiply both sides LM by f

\[ fr = eY - \frac{M}{P} \]

if \ f = 0, \ Y = \frac{1}{e} \frac{M}{P} \\

Therefore \ \text{LM is vertical} \ (Y \ does \ not \ depend \ on \ r).

AD \ \ Y = \frac{1}{e} \frac{M}{P}. \ \text{The AD curve is a rectangular hyperbola}, \ for \ a \ given \ M. \ AD \ does \ not \ react \ to \ fiscal \ policy. \ Only \ to \ monetary \ policy.

c) \ Money \ demand \ does \ not \ depend \ on \ Y \ (e=0) \\

If e=0, z=1
LM \quad r = - \frac{1}{f} \frac{M}{P}. \text{ Therefore LM is horizontal; does not depend on } Y.

\begin{equation}
\text{AD } Y = \frac{a + c}{1 - b} + \frac{1}{1 - b} G - \frac{b}{1 - b} T + \frac{d}{(1 - b) f} \frac{M}{P}
\end{equation}

Therefore, AD shifts to fiscal policy according to full multipliers. Fiscal policy is very effective.

\textit{d) Money demand is very sensitive to } r

If \( f \) very large, \( z \to 1 \)

\textit{LM is horizontal.} This can be seen from the formula of LM.

\begin{equation}
\text{AD } Y = \frac{a + c}{1 - b} + \frac{1}{1 - b} G - \frac{b}{1 - b} T. \text{ Therefore AD is vertical. } Y \text{ does not depend on } P. \text{ Fiscal policy fully effective.} 
\end{equation}
9. a) Expansive monetary policy ($\Delta M$) to lower interest rate and raise investment, and contractionary fiscal policy ($\Delta T$) to lower consumption and compensate the effect of the expansive monetary policy on output. This will lower the interest rate, increase investment, decrease consumption and generate public surplus.

In the US the policy followed in the early 80’s was exactly the opposite: tight monetary policy Shift upwards of the LM curve), which meant a substantial increase in the rate of interest and therefore a fall in investment, and a lax fiscal policy
(shift rightwards of the IS curve), by reducing taxes, which raised consumption and generated a public deficit.

10. a) Shocks come from the IS curve. Then better to hold constant $M$ and let $r$ adjust.
By holding $M$ constant the Central Bank steers the economy from A to B. Whereas if it had followed an accommodating monetary policy to keep the interest rate constant, it would have led the economy to A’, with a much larger oscillation in output.

b) Shocks come from the LM curve. Then better to counteract these shocks by a monetary policy that maintains constant the interest rate.

11. If the demand for money depends on disposable income, the LM curve takes the following form:

$$r = \frac{e}{f} Y - \frac{e}{f} T - \frac{1}{f} \frac{M}{P}$$

This means that movements in taxes will not only move the IS curve, but also the LM curve.
a) The effects of, say, raising government expenditure will, as in the basic model, move only the IS curve to the right.

b) The effects of lowering taxes will move both the IS curve (to the right) and the LM curve (upwards). This leads to an increase in the interest rate, and may lead to a fall in income (or in any case, a smaller increase than in the basic model)